Efficient, Realistic, Physics-based Modeling for Buried UXO Based on Time-domain Electromagnetic Scattering Signatures

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The aim of this effort was to deliver to SERDP a software product suitable for transition to time-domain electromagnetic (TDEM) based sensors currently used for UXO discrimination. The methodology was intended to simultaneously address the requirements of (i) high fi-delity physics-based modelling for realistic target shapes and (ii) vastly accelerated CPU efficiency for forward modelling and inversion, and subsequent discrimination. This aim has essentially been achieved, as described below, through continued development of our highly effi- cient mean field approach, together with the development of an entirely new complementary early time approach, to high contrast EM scattering. This Final Report will summarize the substantial advances we have made, describe the nature of the work that still needs to be done to turn our results into a final self-contained product; and finally describe our ongoing and planned future efforts to transition that product to laboratory and field use.

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Final Report to SERDP for project UX-1311

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(Dated: August 19, 2005)

This document summarizes all work performed under SERDP project UX-1311, "Efficient, realistic, physics-based modelling for buried UXO based on time-domain electromagnetic scattering signatures," referring to accompanying supplementary reports for details. Ongoing plans and activities to transition the modelling technologies developed under the project to future laboratory and field applications are also described.

I. INTRODUCTION

The aim of this effort was to deliver to SERDP a software product suitable for transition to time-domain electromagnetic (TDEM) based sensors currently used for UXO discrimination. The methodology was intended to simultaneously address the requirements of (i) high fidelity physics-based modelling for realistic target shapes and (ii) vastly accelerated CPU efficiency for forward modelling and inversion, and subsequent discrimination. This aim has essentially been achieved, as described below, through continued development of our highly efficient "mean field" approach, together with the development of an entirely new complementary "early time" approach, to high contrast EM scattering. This Final Report will summarize the substantial advances we have made, referring to the attached publications and technical reports (Refs. [1–8, 10] listed in the bibiliography) for details; describe the nature of the work that still needs to be done to turn our results into a final self-contained product; and finally describe our ongoing and planned future efforts to transition that product to laboratory and field use.

	Intermediate and late time	Early time
Underlying theory	Mean field theory [1, 2, 6]	Early time theory [3–5]
Intrinsic dynamical entities	Bulk modes	Surface modes
Mode computational algorithm	Mean field internal code [7]	Surface mode code [5]
Extrinsic (measurement prediction) algorithm	Mean field external code [8]	External surface mode code [5]

TABLE I: Summary of how the UXO modelling problem may be divided into computation of intrinsic features (modes) and extrinsic features (measurement predictions), using algorithms designed to address the two essentially different regimes of the time domain electromagnetic response of compact, highly conducting targets. Citations are also given to attached publications and reports where detailed descriptions may be found.

A. Basic modelling strategy: intrinsic vs. extrinsic features

In order to develop a rapid numerical EM modelling capability one must take optimal advantage of several key features of the UXO discrimination problem, namely the compact, highly conducting, roughly spheroidal physical characteristics of UXO targets. These allow one to separate, as summarized in Table I, the solution algorithms (appropriate to each dynamical regime, as described below) into "slow" and "fast" parts [1]. The slow algorithm may be run in advance, generating as output a database of key *intrinsic* EM signatures for a range of expected targets. These signatures efficiently encode the response of the target to all possible external excitations. The database is of manageable size precisely because of the restricted class of targets. There are also simple scaling laws that further reduce the storage: the responses of geometrically similar targets, with different overall diameter and conductivity, but identical aspect ratios, may be simply mapped onto each other [2]. This precomputed data is then used as input to the fast algorithm, which uses the *extrinsic* characteristics of the measurement platform (transmitter and receiver coil geometry, pulse waveform characteristics, target-platform relative position and orientation, soil characteristics, etc.) to predict the measured data in near real time. The rapid forward prediction is required because solution of the inverse problem, namely discovering the target that best matches the data, involves an iterative process that may require hundreds, or even thousands, of forward computations. A truly useful software tool would require successful inversion in minutes, not hours, hence an individual forward computation should take only a faction of a second. As described further below, we have now achieved this goal, and are presently working on incorporating this capability into our inversion tool.

The identification of the division of labor between intrinsic and extrinsic features, and the great improvement in numerical efficiency that results, lies at the heart of the novelty of our approach. All other physics-based EM codes of which we are aware require independent runs for each new realization of the initial data (e.g., each new target/instrument position or orientation).

B. Physical regimes: early, intermediate and late time

The intrinsic and extrinsic computations each further divide into early time (or, equivalently, high frequency in a frequency domain measurement) and intermediate-to-late time (or intermediate-to-low frequency) regimes, reflecting the entirely different physical nature of the target electrodynamics in the two regimes. At early time, following a rapidly terminated transmitted pulse, a set of screening currents are induced on the surface of the target, and their initial diffusive penetration into the target dominates the measured response, which exhibits a characteristic $t^{-1/2}$ or $t^{-3/2}$ power law singularity in the induced voltage, depending on the magnetic characteristics of the target [3–5]. The intermediate time regime begins as these currents penetrate a substantial fraction of the target diameter into the interior, and the primarily surface response crosses over to a volume response characterized by a superposition of exponentially decaying modes [1, 2]. At late time only the slowest decaying mode survives and the response is a pure exponential.

Corresponding to these two regimes, there are two entirely different theories, based on rigorous solutions to the underlying Maxwell equations, to describe them, each with its own numerical implementation. The *mean field approach* to the intermediate-to-late time regime is summarized in Sec. II, and more details can be found in the attached reports and publications, Refs. [1, 5–8]. The *early time approach* is summarized in Sec. III, and more details can be found in the attached reports and publications, Refs. [3–5].

To summarize, our modelling approach has the great advantage that it clearly exhibits the underlying *mathematical structure* of the predicted data (in terms of early time surface modes and mean field bulk modes), making it easy to identify and interpret key aspects of the EMI signature. In particular, by specifying the precise way in which the intrinsic and extrinsic target properties contribute to the final signal, all of the guess work is taken out of the data analysis, and a very direct, intuitive evaluation of the discriminatory content of the measured data emerges.

C. Standardized excitation approach

Since it is actually part of our ongoing work, to close this introductory section it is worth comparing and contrasting our methodology with the recently developed standardized excitation approach (SEA) [9]. The strategy for the latter is, in many ways, similar to ours. The SEA also relies on the creation of a data base of intrinsic EM signatures for a given target, from which the response to a general excitation can be rapidly computed. However, rather than computing such signatures from first principles, they are instead derived from series of laboratory measurements on the target in question. The most recent formulation of the SEA relies on a comprehensive set of frequency-domain laboratory measurements (sweeping the frequency, and using varying excitation field, target position and orientation, etc.) to extract the target UXO response to a fundamental set of excitations (uniform, linearly varying, quadratically varying, etc., applied magnetic field in the target neighborhood), which may then be superimposed to compute the response to a general excitation [9].

Thus, let the frequency-domain transmitted (primary) magnetic field, in the *absence* of a target, be decomposed in the form

$$\mathbf{H}^{\rm pr}(\mathbf{x},\omega) = \sum_{j=1}^{\infty} b_j \mathbf{h}_j^{\rm se}(\mathbf{x}) \tag{1.1}$$

in which the origin lies at the target center, and $\mathbf{h}_j^{\rm se}(\mathbf{x})$ form a complete set of basis functions which are regular at the origin, for example those obtained from a spherical or spheroidal harmonic expansion of the magnetic potential in the neighborhood of the origin [9]. The coefficients b_j encode the characteristics of the transmitter loop, and are readily calculated. In most applications the frequency is low enough that the environment may be treated as effectively insulating, and both b_j and $\mathbf{h}^{\rm se}$ are frequency independent. Linearity of the Maxwell equations allows one to infer the scattered field in the presence of a target in the form

$$\hat{\mathbf{H}}^s(\mathbf{x},\omega) = \sum_{j,k=1}^{\infty} b_j S_{jk}(\omega) \mathbf{h}_k^s(\mathbf{x}) \tag{1.2}$$

in which $\mathbf{h}_k^s(\mathbf{x})$ form a complete set of basis functions that are regular at infinity (derivable, for example, from a spherical or spheroidal harmonic expansion [9]), and the coefficients $S_{jk}(\omega)$ give the response to the isolated standard excitation j. Thus, if one knows the response to each individual b_j, an arbitrary excitation may be constructed as a linear superposition of these responses.

Although $\mathbf{h}_k^s$ is also frequency independent in an effectively insulating environment, the frequency dependence of S_{jk} is crucial since it represents the electrodynamics of the target. It is these coefficients that are used to uniquely characterize a particular target, and comprise its element of the database. They are determined by performing a series of linearly independent measurements of the left hand side of (1.2) at different positions and frequencies, and then finding the optimal values of (a subset of) the S_{jk} that best reproduce the data.

For time-domain applications, the frequency coverage of the measurements must be broad and dense enough for their Fourier transform to capture the three different regimes described above. To accurately represent the results of a given measurement, the number of standardized excitations must be high enough to account for the full structure of the transmitted field. In general, the complexity of the excitation increases in the near field, i.e., as the target-instrument separation decreases, and

the required number of S_{jk} needed to model such a measurement would increase.

Clearly our model-based and the data-assimilation-based SEA each have characteristic advantages and disadvantages. The essential differences between the two may be summarized as follows:

1. SEA in principle provides exact responses for a given physical target, whereas the modelling approach is restricted to a somewhat idealized class of ellipsoidal geometries.

2. On the other hand, the SEA can only be used to characterize a target for which detailed laboratory measurements have been made in advance, whereas, within the allowed class of geometries, the modelling approach allows new targets to be added to the data base relatively quickly.

3. The modelling approach clearly exhibits the detailed underlying physics of a given target response, whereas the SEA basically treats the target as a "black box," making no use of the underlying Maxwell equations beyond their linearity.

4. The SEA is limited in that, in essence, only previously performed measurements can be modelled. Each new situation, that drives the target into a previously unexplored regime, requires a new parameterization of measured data. The modelling approach, on the other hand, is limited by one's ability to compute new responses, not by one's ability to measure them. It is certainly possible to perform laboratory measurements that are beyond the capacity of the present numerical codes to model (e.g., by placing very complicated transmitters very close to the target so as to induce very rapidly varying spatial current distributions), but under normal circumstances we have found that the modelling approach generates a data base with a much more comprehensive set of target responses than does the SEA.

We are presently involved in an Army funded collaboration with the CRRELL group to, among other things, compare in greater detail the utility of the two methods under field conditions.

II. SUMMARY OF MEAN FIELD APPROACH

The mean field approach expresses the electric field, and corresponding EMI voltage, as multi-exponential mode sums,

$$\begin{aligned} \mathbf{E}(\mathbf{x},t) &= \sum_{n=1}^{\infty} A_n \mathbf{e}^{(n)}(\mathbf{x}) e^{-\lambda_n t} \\ V(t) &= \sum_{n=1}^{\infty} V_n e^{-\lambda_n t} \end{aligned} \tag{2.1}$$

in which λ_n are the decay rates, $\mathbf{e}^{(n)}(\mathbf{x})$ are the mode shapes, and A_n are the excitation amplitudes. The Maxwell equations may be reduced to an eigenvalue equation [1, 2],

$$\nabla \times \frac{1}{\mu} \nabla \times \mathbf{e} = \frac{4\pi\sigma}{c^2} \lambda \mathbf{e} \tag{2.2}$$

for the $\lambda_n, \mathbf{e}^{(n)}$, in terms of the spatially dependent permeability μ and conductivity σ. These represent precisely the *intrinsic*, measurement independent features discussed above.

A. Internal code

The "slow" part of the mean field algorithm is focused on solving the eigenvalue equation (2.2) numerically and storing the resulting mode shapes and decay rates. A detailed description of this part of the code, along with the theory underlying each section, is contained in the attached Ref. [7]. While the mode shapes are nonzero both inside and outside the target, the algorithm is based on the integral equation version of (2.2), which allows one to reduce the latter to an equation for the internal field alone. The internal values are stored in the form of the coefficients in the Taylor expansion of $\mathbf{e}^{(n)}(\mathbf{x})$, with the accuracy of the calculation determined by the maximum allowed polynomial order n of the terms. While the integral equation allows one to derive the the external field from the internal field via a volume integral of the Green function over the target, this is a numerically inefficient process. However, not too close to the target (where most measurements in fact take place), the external values may be accurately represented as a multipole expansion whose coefficients may be precomputed and stored as well.

For reasonable values $n \le 7$, the numerical approach permits accurate computation of the first several dozen slowest decaying modes, and hence allows rapid prediction of the intermediate- to late-time signal. Here, at late time, $t > 1/\lambda_1$, only the slowest decaying mode survives, while the intermediate time interval, $1/\lambda_{\max} < t < 1/\lambda_1$, is bounded on the left by the inverse of the largest computed decay rate $\lambda_{\max}$. It is precisely this estimate which invalidates the approach at early time: as $t \to 0$ an essentially infinite set of modes, far beyond one's ability to compute, contributes to the response.

B. External code

The excitation coefficients A_n, V_n are the *extrinsic* features, depending sensitively on the transmitter and receiver loop geometries, and the relative position and orientation of the target and measurement platform. The mode excitation coefficient A_n may be expressed in terms of a line integral around the transmitter loop C_T:

$$A_n = I_T \oint_{C_T} \mathbf{e}^{(n)}(\mathbf{x})^* \cdot d\mathbf{l} \tag{2.3}$$

where the total current $I_T = i_T N_T$ is the product of the actual transmitter current with the number of windings. The voltage amplitudes V_n require a further line integral around the receiver loop C_R:

$$V_n = A_n N_R \oint_{C_T} \mathbf{e}^{(n)}(\mathbf{x}) \cdot d\mathbf{l}, \tag{2.4}$$

where N_R is the number of receiver loop windings.

The "fast" part of the mean field algorithm uses the stored mode data to compute the integrals (2.3) and (2.4). A detailed description of the code, together with the underlying theory, is contained in the attached Ref. [8]. Both require multiple evaluations of the external field, and the code provides the option of either doing this exactly through the integral equation, or approximately via the multipole expansion. It has been found that for simulations of realistic measurement scenarios, the latter provides more than sufficient accuracy, allowing the measurement predictions, as alluded to earlier, to be made in a fraction of second. Illustrative applications of both the internal and external codes may be found in the attached Ref. [1].

III. SUMMARY OF COMPLEMENTARY EARLY TIME APPROACH

The breakdown of the mean field algorithm at early time is a direct reflection of the very rapid electrodynamics that follows the rapidly terminated transmitted pulse. The complementary early time approach accounts for the fact that as $t \to 0$ an essentially infinite number of terms contribute to the mode sum. We have developed a novel method, described in detail in the attached Refs. [3–5], valid for $t \ll 1/\lambda_1$, that effectively resums this series. The electric field is now expressed as a sum over *surface modes*, obtained from a solution of a corresponding surface eigenvalue equation [4]. The mode excitation amplitudes are determined by the pattern of screening currents generated on the target surface by the rapidly terminated transmitter pulse, and may be computed from the transmitted field just prior to pulse termination. Computing the latter actually requires solving for the electrodynamics of the target during the more gradual turn-on time of the pulse. The latter may be computed using the mean field algorithm, which therefore produces an interesting extra level of synergy between the two sets of codes: the mean field code provides both the initial data for the early time code, and the extension of the solution out of the early time regime.

The basic result of the early time theory is that EMI voltage may be expressed in the form of a series [4]

$$V(t) = \sum_{n=1}^{\infty} V_n^{(e)} \left[\frac{1}{\sqrt{\pi t}} - \kappa_n e^{\kappa_n^2 t} \mathrm{erfc}(\kappa_n \sqrt{t}) \right] \tag{3.1}$$

in which $V_n^{(e)}$ are early time excitation amplitudes, and κ_n are surface eigenvalues. The first term displays a divergence, $V(t) \propto t^{-1/2}$, at very early time $\lambda_1 t \ll (\mu_b/\mu_c)^2$ (reflecting the initial diffusion of the currents into the target). The second error function term leads to a crossover to a steeper decay, $V(t) \propto t^{-3/2}$, for $(\mu_b/\mu_c)^2 \ll \lambda_1 t \ll 1$. Here μ_b and μ_c are the background and target permeability, respectively. For ferrous targets, where $\mu_c/\mu_b = \mathcal{O}(10^2)$, only the latter regime is visible. Both regimes are evident in experimental data: aluminum targets display a clear $t^{-1/2}$ regime, while steel targets display a broad $t^{-3/2}$ regime. For larger ferrous targets the early time regime may encompass the entire measurement window of the instrument—one may never even see the intermediate time multi-exponential behavior. The ubiquity of ferrous UXO underscores, therefore, the critical importance of accurate early time modelling.

Given the unexpected complexity of the underlying theory, and the extra time required to develop it, its numerical implementation, although considerably simpler, has not advanced as far as that of the mean field approach. We have developed pieces of code that implement most aspects of the early time predictions, but have not yet assembled them into a comprehensive, user-transparent form. This is part of our ongoing work described in Sec. V below.

IV. INVERSION ALGORITHM

The ultimate purpose of the forward modelling codes is as input to an inversion algorithm that uses them to discover the target that best predicts the measured signal. Rapid inversion (a few minutes at most) requires extraordinarily rapid forward computations (fractions of a second at most), which we have now achieved. It is worth emphasizing the advantage of having full forward predictions, including both intrinsic and extrinsic effects. The only intrinsic features contained in the voltage series (2.1) are the decay rates λ_n, of which, optimistically, at most two or three might be extracted from noisy data. This would not provide nearly enough information for reliable inversion. It is far better to generate a prediction $V_p(t)$ for the entire voltage curve, which can be directly compared to measured data, for example through the error function

$$E = \sum_{i=1}^{N_t} [V_p(t_i) - V_i]^2, \tag{4.1}$$

in which V_i are the measured voltages at the time gates t_i, $i = 1, 2, \ldots, N_t$.

We have implemented precisely this strategy using a genetic algorithm-based inversion code to search over the stored database of targets to find the one that minimizes the error E. The search parameters include not only the target geometry, but also its position and orientation. In the attached Ref. [1], examples are shown using noise-corrupted simulated data generated by the mean field algorithm using a data base of spheroids with a broad range

of aspect ratios. Unfortunately, lacking a proper merger of the mean field and early time codes, we are at this stage only able to perform inversions on intermediate-to-early time data. We are presently working on remedying this deficiency so that time-domain predictions can be made and compared to data spanning the full dynamical range.

V. TRANSITION PLANS AND ACTIVITIES

We close by summarizing our ongoing and future plans to improve our codes and transition them for use to other groups in the UXO remediation field, especially those tied to SERDP/ESTCP programs. With future applications in mind, our numerical codes have, throughout their development, been designed to interface with existing EM aircoil and magnetosensor array tools. Our codes already contain detailed specifications (transmitter and receiver loop dimensions, orientation and relative position; transmitter current waveform; receiver time gates; etc.) for a wide variety of TDEM instruments (Geonics, MTADS, NVE MagnetoCube, etc.), and are easily augmentable to new configurations as they become available.

We are presently involved in two UXO-related efforts. The first is a SERDP-sponsored collaboration with Eric Miller at Northeastern University (NEU) to analyze laboratory EMI data and augment his own data analysis tools with our physics-based models. The second is an Army sponsored collaboration with Kevin O'Neill's group at CRRELL to aid in the development of a flexible, comprehensive time-domain electromagnetic (TDEM) instrument simulation tool, to include (1) a combination of our physics- and their laboratory data-based (SEA) EM models for a range of target UXO; (2) a range of realistic statistical models of soil, ground surface, and metallic clutter variability and their contribution to the background EM signal; and (3) accurate instrument models, including transmitter and receiver loop geometries, pulse waveform characteristics, and calibration factors for accurate measurement interpretation. These are to be combined so that accurate forward predictions of measured TDEM signals, and their intrinsic variability, can be made under realistic background conditions. The finished product is intended to allow one to simulate expected data collected under specified conditions appropriate to a given UXO remediation site. It could also serve as an input to an inversion algorithm which finds the target whose predicted signal best matches the collected data.

The following is a list of possible future directions, some of which are actively being pursued, others of which remain on the table. In the case of the former, we note the relevant collaboration.

1. **Complete merger of early-time an mean field codes:** Properly assemble the early time codes into a complete package, and unify them with the mean field codes. This is presently being pursued as part of our NEU collaboration.

2. **Expand target geometry database:** Generalize the codes to deal with more general classes of non-ellipsoidal targets. We have mainly dealt so far with solid spheroidal targets (both ferrous and non-ferrous) for which we have large mode databases. We also have developed the capability of dealing with more general triaxial ellipsoids and hollow ellipsoids, but have not thoroughly tested the limitations of the algorithms for such targets. Along with establishing our ability to deal with such geometries, we would further like to study asymmetric targets with more UXO-like shapes.

3. **Inhomogeneous backgrounds:** Generalize the codes to deal with horizontally stratified backgrounds, especially permeable backgrounds. The theory has been worked out in full detail (see the attached Ref. [10]), but not yet numerically implemented. It has been found, for example, that even very small soil permeability contrast (less than 1%) leads to measurable changes to the background signal. We will be pursuing this work under the CRRELL collaboration.

4. **Multiple and composite targets:** UXO remediation sites often consist of areas with dense metallic clutter (bullet casings, shrapnel, exploded bomb parts, etc.) that can interfere with detection of the UXO of interest. Targets not too close together (and this is often not a very strong requirement) can be treated independently, contributing to a measurement via simple superposition. Such situations would be very easy to implement, and together with an appropriate Monte Carlo algorithm implementing an appropriate probability distribution, would allow one to characterize the background signal variability due to such clutter. Time permitting, such an investigation will be part of the CRRELL collaboration.

   Composite targets, or multiple targets in close proximity, are electromagnetically coupled, and would have to be modelled as a single unit. The underlying theory of such coupled systems has already been developed [6], but has not yet been numerically implemented. The ability, especially, to model composites would allow one to further improve the fidelity of fit to real UXO.

5. **Optimal survey and instrument design:** Apply our modelling tools to practical issues of optimal survey and sensor design. We would like to develop a methodology for determining optimal survey design for existing platforms (Geonics, Geophex, MTADS, etc.), as well as determining optimal designs for new instruments (with innovative transmitted field geometries, and multi-axis, multi-modal sensor arrays), by minimizing appropriate objective functions (which balance desired inversion fidelity with practical and engineering con-

straints, such as signal-to-noise ratio, bandwidth, survey time and cost, and ease of operation), averaged over an ensemble of realistic target and clutter scenarios. Such optimization studies would have to be performed in close consultation with our contacts in the UXO remediation industry, leading to survey and instrument design strategies that can be subsequently validated through field demonstrations.

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